STATISTICAL PROPERTIES OF FRAGMENT ISOSPIN IN MULTIFRAGMENTATION

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Abstract

It is shown that the chemical equilibrium condition of the system can be unambiguously identified by analyzing the isospin evolution of fragments produced in nuclear multifragmentation process. As far as the chemical equilibrium is established, the isotope production can be used for finding properties of nuclear matter and fragments in the freeze-out volume, for example, via the isoscaling phenomenon.

1 INTRODUCTION

Isotopic effects and chemical equilibrium in nuclear reactions are receiving increasing attention because they give a new insight into the nuclear matter properties at extreme conditions. This is of high current interest, in particular, for astrophysical applications. Reaction of multifragmentation of excited nuclei is very promising in this respect since it can be considered as manifestation of liquid-gas type phase transition in nuclei, therefore, it can directly address the nuclear matter behavior at subnuclear densities. A number of theoretical and experimental results was already reported on this subject [1–6]. Special interest to this problem arose after discovering the nuclear caloric curve based on the fragment isospin properties [7] and observation of similar isotopic temperatures in many experiments.

2 STATISTICAL APPROACH

It is commonly accepted that the detailed balance principle is responsible for the chemical equilibrium between fragments of different sizes. Within statistical models, this can be written as a relationship between their chemical potentials. In the finite size systems we can directly implement this principle by taking into account all partitions with corresponding weights. For the case of the nuclear multifragmentation, both methods give very close results concerning the fragment isospin.

The statistical multifragmentation model (SMM) is applied for the following analysis. This model has been developing since long ago and is successful in describing experimental data (see, e.g., references in [6, 8]). Presently, many other statistical models are based on similar conceptions, therefore, the demonstrated physical results are representative for the whole statistical approach. The SMM assumes statistical equilibrium of highly excited nuclear systems at a low-density freeze-out stage $\rho \lesssim \rho_0/3$ ($\rho_0 \approx 0.15$ fm^{-3} is the normal nuclear density) [8]. The SMM includes also the low energy de-excitation decay modes and predicts a natural transition to the multifragmentation regime from the compound nucleus decay [9]. All breakup channels (partitions) composed of nucleons and excited fragments are considered and the conservation of mass, charge, momentum and energy is taken into account. In the microcanonical treatment the statistical weight of decay channel j is given by $W_j \propto exp\, S_j$, where S_j is the entropy of the system in channel j which is a function of the excitation energy E_x, mass number A_s, charge Z_s and other parameters of the source. Different breakup partitions are sampled according to their statistical weights uniformly in the phase

space. After breakup, the fragments propagate independently in their mutual Coulomb field and undergo secondary decays. The deexcitation of the hot primary fragments proceeds via evaporation, fission, or Fermi-breakup [10]. At the freeze-out density the SMM treats hot fragments (A, Z are mass number and charge of the fragments) as nuclear matter pieces: Light fragments with $A \leq 4$ are considered as stable particles ("nuclear gas") with masses and spins taken from the nuclear tables. Fragments with $A > 4$ are treated as heated nuclear liquid drops, and their individual free energies F_{AZ} are parameterized as a sum of the bulk, surface, Coulomb and symmetry energy contributions

$$F_{AZ} = F_{AZ}^B + F_{AZ}^S + E_{AZ}^C + E_{AZ}^{sym}. \quad (1)$$

The standard expressions [8] for these terms are: $F_{AZ}^B = (-W_0 - T^2/\epsilon_0)A$, where the parameter ϵ_0 is related to the level density, and W_0=16 MeV is the binding energy of nuclear matter; $F_{AZ}^S = B_0 A^{2/3}(\frac{T_c^2 - T^2}{T_c^2 + T^2})^{5/4}$, where B_0=18 MeV is the surface coefficient, and T_c=18 MeV is the critical temperature of nuclear matter; $E_{AZ}^C = cZ^2/A^{1/3}$, where c is the Coulomb parameter obtained in the Wigner-Seitz approximation, $c = (3/5)(e^2/r_0)(1 - (\rho/\rho_0)^{1/3})$, with the charge unit e and r_0=1.17 fm; $E_{AZ}^{sym} = \gamma(A - 2Z)^2/A$, where γ=25 MeV is the symmetry energy parameter. These parameters are those of the Bethe-Weizsäcker formula and correspond to the assumption of isolated fragments with normal density in the freeze-out volume, an assumption found to be quite successful in many applications. It is to be expected, however, that in a more realistic treatment primary fragments will have to be considered not only excited but also expanded and still subject to a residual nuclear interaction between them. These effects can be accounted for in the fragment free energies by changing the corresponding liquid-drop parameters, provided such modifications are also indicated by the experimental data. For example, for the symmetry energy, this information may be obtained from the isoscaling

phenomenon. An unresolved problem for all statistical models on the marked is the flow development observed at very high excitation energies. However, it seems not crucial for the isospin studies, since the flow influences mainly the kinetic energies of fragments and, possibly, their charge distributions. As far as the charge distributions are described, the isotope composition of the fragments can be reliably addressed within the statistical approach.

3 EVOLUTION OF THE FRAGMENT ISOSPIN

The crucial questions to be answered are: 1) what is the isospin distribution for fragments with fixed A (or Z), 2) how does the isospin of fragments change with the isospin of the thermal source, 3) how does the fragment isospin change with excitation energy of the source, when the fragment charge distribution evolves. In the grand canonical approximation [9], the mean multiplicity of fragments is given by

$$\langle N_{AZ} \rangle = g_{AZ} \frac{V_f}{\lambda_T^3} A^{\frac{3}{2}} \exp\left[-\frac{F_{AZ} - \mu A - \nu Z}{T}\right], \quad (2)$$

where g_{AZ} is the degeneracy factor, λ_T is the nucleon thermal wavelength, V_f is the "free" volume, and μ and ν are the chemical potentials responsible for the mass and charge conservation in the system, respectively [8, 9]. As shown in Ref. [10], the average charge $\langle Z_A \rangle$ of fragments with mass A and the width σ_Z^A of their charge distribution can be written as

$$\langle Z_A \rangle \simeq \frac{(4\gamma + \nu)A}{8\gamma + 2cA^{2/3}}, \quad \sigma_Z^A \approx \sqrt{\frac{AT}{8\gamma}}. \quad (3)$$

The symmetry energy, Coulomb interaction and the chemical potential ν are directly responsible for the isospin of the produced fragments. As seen from Eq. (3), the neutron content of the fragments increases with their mass number, because of the Coulomb term. This trend exists in the stable nuclei and it presents also in multifragmentation experimental data. One can see it from Fig. 1, where the neutron
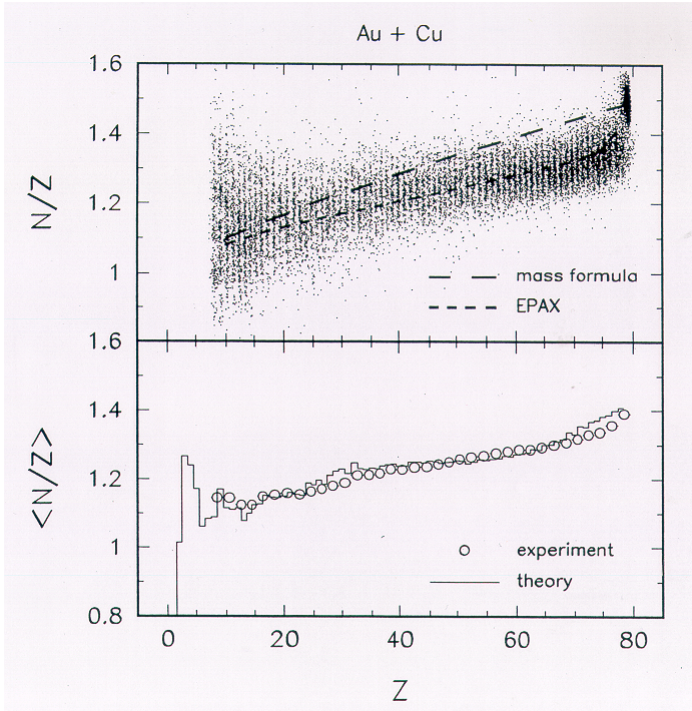
Figure 1: Measured N/Z (top) and mean value $\langle N/Z \rangle$ (bottom) as a function of the charge Z of projectile fragments from the reaction ^{197}Au + Cu at E/A=600 MeV. The lines in the top panel represent the valley of stability according to the Weizsäcker mass formula (long dashed) and EPAX parameterization [11] (short dashed). In the bottom panel, the experimental data are given by the open circles and the prediction of the SMM by the histogram.
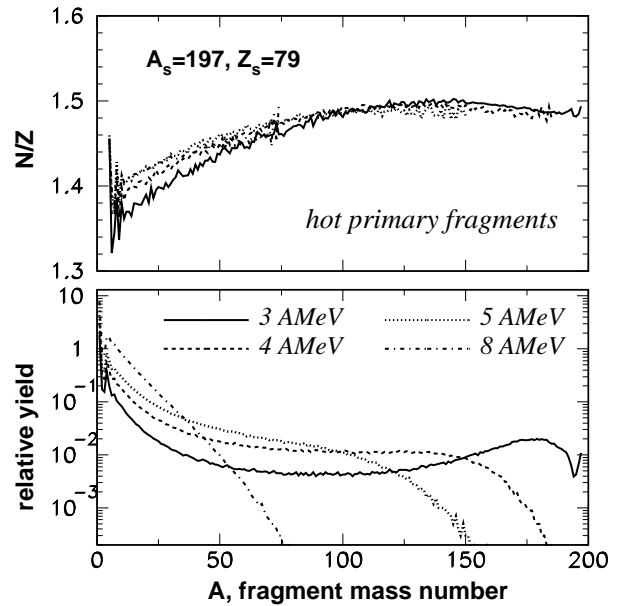

Figure 2: The neutron-to-proton ratio N/Z (top) and relative yield (bottom) of hot primary fragments produced after the break-up of Au nuclei at different excitation energies: 3 (solid lines), 4 (dashed lines), 5 (dotted lines) and 8 (dot-dashed lines) MeV per nucleon.

to proton (N/Z) ratio of fragments observed by ALADIN [12] at multifragmentation of Au-like sources is shown. The agreement of the SMM calculations with the data is obtained simultaneously with a very good description of the fragment partitions [12], this is an important requirement for this kind of analysis.

An intuitive expectation that the fragments produced from a neutron rich source are also neutron rich was confirmed in many experiments (see, e.g., [3, 5]). It is fully consistent with the statistical predictions: As calculations show [4, 6] the chemical potential ν decreases with the neutron excess of the source, therefore, this effect for intermediate mass fragments (IMF, charges Z=3–20) is explained by Eq. (3).

In Fig. 2 the N/Z ratio of the fragments together with their mass distributions are shown. The calculations were done with the micro-canonical Markov-chain SMM version [4] to take into account the mass and charge conservation explicitly. The results are consistent with the grand canonical ones (see also [6, 8]), however, one can see that the finite-size effects favor the saturation of the N/Z ratio of very large fragments with $A > A_{\mathrm{s}}/2$. This deviation from Eq. (3) is important for low multiplicity channels at small E_{x}.

The SMM predicts an interesting behavior of fragment isospin with changing fragment charge distribution. In Fig. 2 one can also see the evolution of the N/Z ratio and mass distribution of fragments in the excitation energy range E_{x}=3–8 MeV/nucleon. It is seen that the fragment mass distribution evolves from the U–shape, at the multifragmentation threshold E_{x}=$E_{th} \approx 3$ MeV/nucleon, to an exponential fall-off at high energies. During this

evolution the temperature reaches a "plateau" and is nearly constant [7, 8]. As the energy increases the N/Z ratio of primary IMFs' increases, too. The reason is that the heaviest neutron-rich fragments are destroyed at increasing excitation energy, and most of their neutrons are bound in IMFs, since the number of free neutrons is still small at this stage. Generally, the N/Z ratios of fragments decrease after their secondary deexcitation (e.g., compare Fig. 1 and Fig. 2). If the temperature increases considerably with the excitation energy (in the energy region of small $E_x < E_{th}$) the N/Z ratios of final cold fragments can decrease with the energy. However, when the temperature reaches the "plateau" the internal excitation energy of hot fragments does not change (see Eq. (1)) and the final isotope composition becomes proportional to the primary fragment isospin. In this case, the effect of increasing the N/Z ratio of IMFs can survive after the secondary deexcitation. The experimental data [3] show that this effect exists and is consistent with the SMM predictions. Therefore, it is a good tool for probing the freeze-out conditions and verification of the chemical equilibrium.

It is important that the demonstrated isospin evolution takes place in the energy range which is usually associated with a liquid-gas type phase transition in finite nuclei [8]. The realistic statistical model, designed to reproduce the experiment, does not support conclusions of some mean-field approaches (see Ref. [13]) about separation of the nuclear matter into a neutron-rich gas and isospin symmetric "liquid" matter during the phase transition. On the contrary, the chemical equilibrium in finite nuclear systems suggests few free neutrons and neutron-rich big fragments at the freeze-out stage. That is consistent with experimental observations.

4 ISOSCALING

By using the chemical equilibrium conception as a guideline one can extract more information about fragments and nuclear matter properties from the analysis of experimental data. The scaling properties of cross sections for fragment production with respect to the isotopic composition of the emitting systems were studied long ago in light-ion induced reactions [2]. Recently, the isoscaling was also reported for heavy-ion reactions [5]. It is constituted by the exponential dependence of the production ratios R_{21} for fragments with neutron number N and charge Z in reactions with different isospin asymmetry:

$$R_{21} = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \cdot exp(N \cdot \alpha + Z \cdot \beta), \quad (4)$$

with three parameters C, α and β. Here Y_2 and Y_1 denote the yields from the more neutron rich and the more neutron poor reaction system, respectively. In some reactions, the parameters α and β have the tendency to be quite similar in absolute magnitude but of opposite sign [5]. This suggests an approximate scaling with the third component of the isospin $t_3 = (N - Z)/2$:

$$R_{12} = \frac{Y_1(N, Z)}{Y_2(N, Z)} = C \cdot exp(-t_3 \cdot \beta_{t3}). \quad (5)$$

As an example, the isotope ratios measured in five pairs of reactions induced by light particles from protons of 0.66 GeV to α-particles of 15.3 GeV incident energy on ^{112}Sn and ^{124}Sn targets, and normalized with respect to the ratio for ^{6}Li, are shown in Fig. 3 (see Ref. [6]). The parameter β_{t3} decreases from 1.08 to 0.68 with increasing energy. The inverse scaling parameter $1/\beta_{t3}$ is found to increase approximately in proportion to the isotope temperature deduced from the yields of He and Li isotopes.

The isoscaling phenomenon arises naturally in a statistical fragmentation mechanism [6, 14]. As evident from Eq. (2), for two systems 1 and 2 with different total mass numbers (A_1 and A_2) and charges (Z_1 and Z_2) but with the same temperature and density, the ratio of fragment yields produced in these systems is given by Eq. (4) with parameters $\alpha = (\mu_1 - \mu_2)/T$ and $\beta = ((\mu_1 - \mu_2) + (\nu_1 - \nu_2))/T$. In the SMM the isoscaling parameters reflect
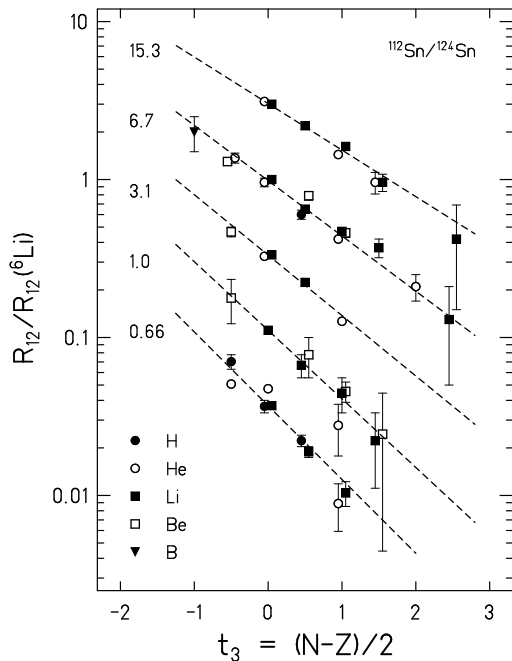
Figure 3: Normalized isotopic effect R_{12} for five reactions, labeled with the projectile energy (in GeV) and shifted by factors of three for clarity. The symbols are for H, He, Li, Be, and B fragments. The lines are the results of the exponential fits according to Eq. (5).

properties of the produced fragments, in particular, the widths of their charge (mass) distributions (see Eq. (3)). As shown in Ref. [6] the potential differences depend essentially only on the coefficient γ of the symmetry term and on the isotopic compositions of the sources but not on the temperature:

$$\mu_1 - \mu_2 \approx -4\gamma(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2}),$$
$$\nu_1 - \nu_2 \approx 8\gamma(\frac{Z_1}{A_1} - \frac{Z_2}{A_2}). \qquad (6)$$

This formula is a very good approximation (within 3%) in the grand-canonics, and it was confirmed for the multifragmentation region ($T \gtrsim 5$ MeV) with the microcanonical Markov chain SMM calculations. Moreover, since the temperature can be found in an independent way, e.g. as the isotope temperature, Eq. (6) provides an effective way for determination of the symmetry energy coefficient γ via experimental α and β parameters.
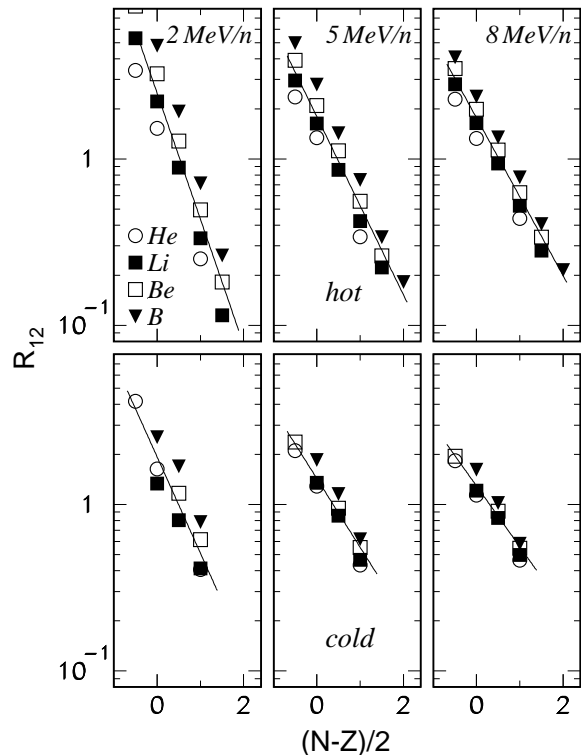

Figure 4: Ratios of isotope yields produced at the breakup of ^{112}Sn and ^{124}Sn sources from Markov-chain SMM calculations for three excitation energies $E_x/A = 2$, 5 and 8 MeV. The top and bottom panels are for hot and cold fragments, respectively. The solid lines give the logarithmic slope parameters β_{t3}=1.74, 1.23, 1.09 for hot fragments, and 1.33, 0.95, 0.84 for cold fragments.

A typical isoscaling obtained with the SMM is shown in Fig. 4. These calculations can be used to estimate quantitatively the influence of the secondary deexcitation on the observable parameters. The isoscaling parameters can change, and the corresponding correction to the extracted symmetry energy coefficient has to be applied. The full analysis was performed in Ref. [6] and the value $\gamma \simeq 22.5$ MeV was reported, which has a tendency to be smaller than the conventional value. Provided it can be substantiated by other data and analyses, this would indicate that the symmetry part of the fragment binding energy is slightly weaker than that of isolated nuclei. Fragments, as they are formed at breakup, may have a lower than normal density.

5 CONCLUSION

Conception of chemical equilibrium is very fruitful for the theoretical description of multi-fragmentation. The chemical equilibrium can be unambiguously verified by analyzing isospin content of the fragments and its evolution together with the fragment yields. A convincing proof of this condition is an excellent description of the fragment charge partitions and the isotope distribution of fragments obtained in different experiments within statistical models (e.g., see [8, 12]). A remarkable trend characterizing the chemical equilibrium is an increase of the N/Z ratio with the mass number of fragments [4, 10]. This is quite instructive because dynamical processes can lead to the opposite ("decreasing") trend, as shown by M. Colonna and M.Di Toro in [1]. Experimental observations of this trend may be obscured by the secondary deexcitation. However, by studying evolution of the N/Z ratio of fragments with changing their charge distribution one can identify it for sure: The effect of increasing N/Z of IMFs within the energy range of the phase transition, observed for the Au sources [3], can be explained in the case of the "increasing" trend only. The chemical equilibrium conception can also provide a key for explanation of many other phenomena. For example, as shown in [4], the midrapidity emission of neutron-rich IMF in peripheral heavy-ion collisions can be explained as a result of the Coulomb proximity of the sources and the angular momentum influence within the statistical picture of the two (projectile-like and target-like) decaying sources. An important advantage of this conception is that it allows for obtaining direct experimental information about nuclear properties at the breakup. A well-known example is the isotope temperatures, extracted from experimental data [7]. Another example is the isoscaling phenomenon, which is connected with the symmetry energy of nuclear matter. As shown here, the isoscaling gives a new way for probing the symmetry energy term of the fragments in the freeze-out volume. All these observations can be directly used in the astrophysical cases (supernovas, neutron stars), where the chemical equilibrium, is believed, to be established.

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